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| Subject Area | Sensor, Actuator and Communication Systems |


# Skeleton surfaces of the 3D-Hénon map

## Software toolbox for finding and visualizing the most stable points in the parameter space

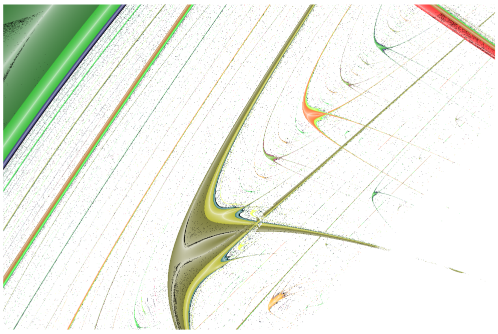

Fig. 1: Parameter space of the 3D-Hénon map. Each color stands for a period between 1 and 50. Color saturation encodes stability of the orbit. Lighter means more stable.

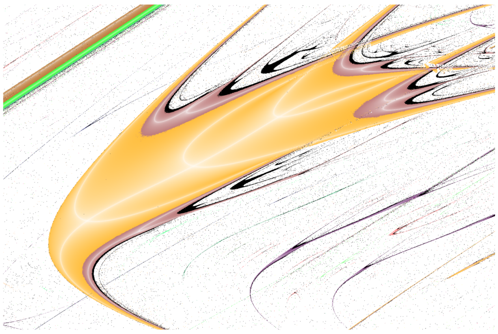

Fig. 2: Zoom to the shrimp of Fig. 1 (below, middle). The set of white points inside the orange shrimp is the skeleton.

Introduction: Nonlinear dynamical systems have been studied for many years. Visualizations of the dependence of dynamical properties on parameters leads to images with a rich structure. A system well known to a general public is the quadratic map, which leads to the famous Mandelbrot set in. The Hénon map shares many dynamical properties with systems important for applications, like models for the behavior of nerve cells. When coloring parameter points according to the period of the corresponding orbit, shrimp-shaped domains can be discerned. Of special interest are orbits of particularly high stability, in two-dimensional pictures, they are represented by light curves within the shrimp-shaped domains.

Objective: In this work a three-dimensional generalization of the Hénon map was studied by extending the two-dimensional techniques. The set of points with particularly high stability, which consisted of curve segments, becomes a set of complicatedly folded surfaces in three dimensions. Since these surfaces span the shrimps, we call them the skeleton. What does the skeleton look like?

Result: The computational power of a cluster of GPUs combined with special software developed during this project allowed to quickly visualize shrimps in two dimensions and the skeleton surfaces as a point cloud in three dimensions. Special techniques were developed to efficiently find points on the skeleton surface. Some steps towards visualizing the surfaces as smooth triangular mesh surfaces were also successful and show promise for further research.

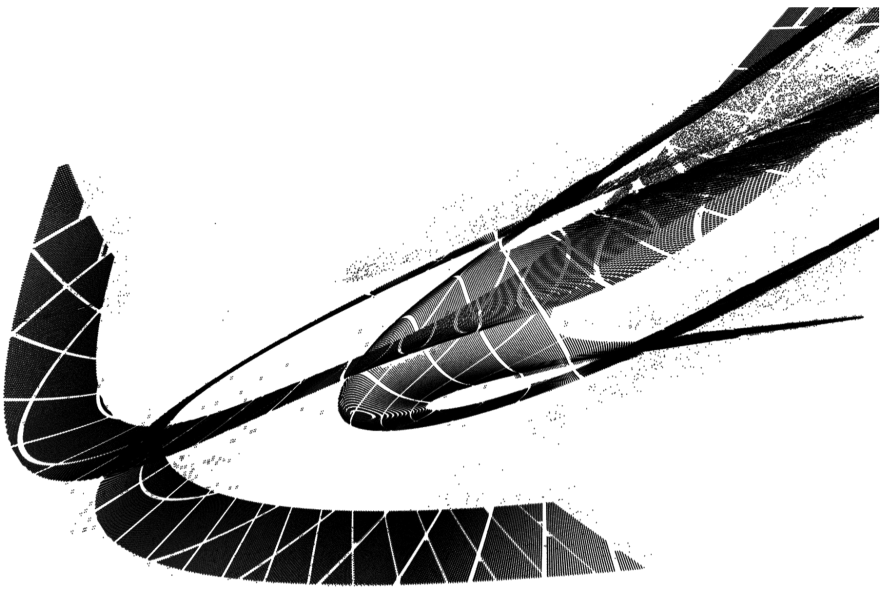

Fig. 3: The set of most stable points as a three dimensional surface, of which Fig. 2 is a two dimensional section.